Engineering Basics of Fusion Plasmas
Reflectometry Diagnostics

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Abstract—Reflectometry diagnostics use electromagnetic waves to measure the electron density profiles of plasmas, being used in fusion devices. The theory of electromagnetic wave propagation in plasmas is detailed for the case of ideal infinite plasmas and in the case of fusion plasmas, such as tokamak plasmas. The ordinary and extraordinary modes are explained and the ordinary mode is presented as the preferred, magnetic field independent mode for reflectometry diagnostics. The data processing mechanisms for the reconstruction of the density profile using an Abel inversion on the reflectometry interference data is presented. Experimental reflectometry results showing the time evolution of density profiles in the occurrence of ELMs are shown.

Index Terms—Reflectometry, Diagnostics

I. INTRODUCTION

Plasma is a quasi-neutral medium of positive and negative particles that interact with the electric and magnetic fields generated by the movement of the remaining particles inside the medium, provoking a collective behaviour effect. Quasi-neutrality is the result that each particle in the plasma interacts mainly with the nearby charges up to a distance called the Debye screening length. This Debye length is typically small compared to the physical size of the plasma medium.

Plasmas exist in low temperature and density conditions, such as in interstellar space, and up to very hot and dense conditions, such as in the centre of stars. These latter conditions result in very energetic particles that allow for nuclear fusion.

Artificial plasmas have been generated in laboratories and used in the industry for decades. Plasmas are used for surface treatment and deposition, semiconductor processing [1] and for fusion power [2], for example. Plasmas discharges can be generated by applying external electric fields. These fields can be continuous or alternate and coupled capacitively or inductively to the plasma [3].

In nuclear fusion, two light atoms are fused together to form a heavier atom, releasing energy, as the resulting mass is less than the original reactant’s total mass. The released energy can then accelerate other atoms and allow a self-sustained nuclear fusion reaction provided there is a source of light atom fuel. From all the possible fusion reactions, the one between deuterium and tritium (D-T) is one of the easiest to attain in laboratory conditions as it has a high reactivity at a lower temperature than in other reactions.

Nuclear fusion reactions occur naturally in the cores of stars. The outwards thermal pressure resulting from the reaction is balanced with the gravity force, in what is called gravity confinement. In order to obtain nuclear fusion in a laboratory setting on Earth, other methods of confinement must be addressed as the energetic particles from the plasma can easily damage or destroy any confinement wall. As we are dealing with charged particles, magnetic fields can be used to confine the plasma and prevent it from running against the walls of the vessel. The tokamak, or Toroidal Kamera and Magnitaya Katushka, is a popular fusion reactor that uses the magnetic confinement concept [2]. The feasibility of fusion energy, by reaching the break-even point where the power output is higher than the power input, is being studied with the development of ITER [4].

The perspective and the vertical cross section views of a tokamak device can be observed in Fig. 1. The toroidal geometry of the tokamak is defined by the distance between the centre of the toroidal chamber and the centre of the plasma column, the major radius $R_0$, and the minor radius $a$ of the plasma column. The inner solenoid acts as the primary transformer circuit while the plasma column forms the secondary. By varying the current in the primary coils, the plasma toroidal current is induced along the tokamak torus. Around the vessel, several poloidal coils generate a toroidal magnetic field, parallel to the plasma current. The combination of the poloidal magnetic field induced by the plasma current and the toroidal magnetic field result in a helical field, a twisted magnetic field around the plasma column, to which the charged particles are trapped.

Fig. 1. Schematic view of a Tokomak. Left: perspective view of the tokomak with relevant component and field descriptions. Right: Cross-section view of the toroidal vessel. [5].

The position and shape of the plasma column can be controlled by varying the external magnetic fields. Controlling the plasma column position is a vital machine protection
function, ensuring that the plasma does not damage the wall. Shape control of the plasma column allows setting the optimal magnetic geometries during the experiment. As magnetic diagnostics provide measurements of fundamental parameters of operation of tokamak plasmas, such as plasma current, loop voltage, position and shape, they are also used for position feedback of the real-time control of the plasma column [6].

Local measurements of the electron density profile in tokamaks are required for the investigation of mechanisms that influence the transport and the plasma confinement. In microwave reflectometry, an electromagnetic wave propagates in the plasma and is reflected at the layer where it equals the local cutoff frequency of the plasma. From the phase shift that the waves suffer and the probing frequency, it is possible to estimate a density profile along the radius. The probing wave can have a fixed frequency, allowing the study of the temporal evolution of the phase, or a broad spectrum to obtain the density profile in the plasma edge [7]. Fast reconstruction of the reflectometry density profiles has allowed the use of this data for plasma position feedback in position control inside tokamak vessels [5].

This paper presents the theory of electromagnetic wave propagation in plasmas and the engineering considerations to take in designing a microwave reflectometry system for plasma density profiles and plasma fluctuation diagnostics. This paper starts by studying the propagation of electromagnetic waves through an ideal plasma medium and in a real tokamak plasma with the presence of magnetic fields. Then the typical microwave reflectometry topology and the measurement principle are presented and explained. Finally, the computation required to reconstruct the density profile from the reflectometry data is detailed.

II. PROPAGATION OF ELECTROMAGNETIC WAVES IN PLASMAS

In this section, the details of the most relevant electromagnetic waves in reflectometry are presented. The information introduced here is based on the study of the propagation of waves in plasmas presented in Introduction to Plasma Theory by Nicholson [8], which should be referred for a more in-depth analysis of wave propagation.

A. Electromagnetic waves in unmagnetized plasmas

Consider an infinite unmagnetized homogeneous plasma and an electromagnetic wave with a high frequency $\omega$ so that $\omega > \omega_{ce}$, where $\omega_{ce}$ is the plasma frequency, and the ions can be considered to be immobile in the timespan of the wave propagation. The relevant Maxwell-fluid equations in Gaussian units are

$$\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}, \quad (1)$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \partial_t \mathbf{E}, \quad (2)$$

$$\mathbf{J} = -e_n \mathbf{V}_e, \quad (3)$$

$$m_e n_e \partial_t \mathbf{V}_e + m_e n_e (\mathbf{V}_e \cdot \nabla) \mathbf{V}_e = -e_n \mathbf{E}, \quad (4)$$

$$\partial_t n_e + \nabla \cdot (n_e \mathbf{V}_e) = 0. \quad (5)$$

Where $\partial_t$ is the time derivative, $\mathbf{J}$ is the current density, $\mathbf{V}_e$ is the electron velocity resulting from the wave’s electric field acceleration, and $\mathbf{E}$ and $\mathbf{B}$ are the wave electric and magnetic fields. It is assumed that the wave is transverse and choose $\mathbf{k}$ so that $\mathbf{k}$ is in the $\hat{z}$-direction, $\mathbf{E}$ is in the $\hat{y}$-direction and $\mathbf{B}$ is in the $\hat{z}$-direction. Here we are assuming a background equilibrium of ions with a small electron perturbation due to the wave. By linearising the system of equations and as the solutions have $k \cdot \mathbf{V}_e = 0$ then $\partial_t n_e = 0$ and $n_e = n_0$. The reduced set of equations then becomes

$$\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}, \quad (6)$$

$$\nabla \times \mathbf{B} = -\frac{4\pi e n_0}{c} \mathbf{V}_e + \frac{1}{c} \partial_t \mathbf{E}, \quad (7)$$

$$m_e n_0 \partial_t \mathbf{V}_e = -en_0 \mathbf{E}. \quad (8)$$

By taking the plane wave solution of this set of equations, one gets the electromagnetic dispersion relation

$$\omega^2 = k^2 c^2 + \omega_{ce}^2, \quad (9)$$

where $\omega_{ce}$ is the electron plasma frequency

$$\omega_{ce}^2 = \frac{4\pi e^2 n_0}{m_e}. \quad (10)$$

Recalling the theory for optical media, the index of refraction travelling through this plasma medium is

$$n = \frac{ck}{\omega} = \sqrt{1 - \omega_{ce}^2 / \omega^2} \quad (11)$$

From index of refraction, it can be seen that for a real $\omega < \omega_{ce}$ the index of refraction becomes imaginary, then $k$ is imaginary, resulting in an evanescent wave for wave frequencies lower than the plasma frequency. It can also be interpreted that an electromagnetic wave that propagates in an inhomogeneous plasma is reflected at the plasma layer where $\omega = \omega_{ce}$, called the position of critical density. This allows the estimation of the density of the plasma layer from the reflected wave frequency, which is the basis of operation of reflectometry diagnostics.

B. Electromagnetic waves in magnetized plasmas

The study of high frequency electromagnetic waves must be extended to magnetized plasmas to fully understand their propagation inside tokamak plasmas, for example. Consider now a wave propagating inside a plasma medium with an external magnetic field $\mathbf{B}_0$ along the $\hat{z}$-direction and the perturbed electric field $\mathbf{E}_1$. A parallel wave propagates along the magnetic field with $k \cdot \mathbf{B}_0 = 1$, whereas a perpendicular wave has $k \cdot \mathbf{B}_0 = 0$. A longitudinal wave has $k \cdot \mathbf{E}_1 = 1$, and a transverse wave has $k \cdot \mathbf{E}_1 = 0$. Consider the first order magnetic field $\mathbf{B}_1$; if $\mathbf{B}_1 = 0$, the wave is electrostatic, otherwise, it is electromagnetic.

The relevant motion equations in magnetized plasmas are the Maxwell-fluid’s together with the electron force equation
used to calculate the current. The linearised set of equations are:
\[ \nabla \times \mathbf{E}_1 = -\frac{1}{c} \partial_t \mathbf{B}_1, \]
\[ \nabla \times \mathbf{B}_1 = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \partial_t \mathbf{E}_1, \]
\[ \mathbf{J} = -e n_e \mathbf{v}_e, \]
\[ m_e n_0 \partial_t \mathbf{v}_e = -e n_0 \mathbf{E} - \frac{en_0}{c} \mathbf{v}_e \times \mathbf{B}_0. \]

Now assume that the waves are travelling perpendicular to \( B_0 \). The electric field can be along \( \hat{z} \), the extraordinary \( x-y \) direction and the \( \mathbf{V} \) on the magnetic field \( \mathbf{B} \). Where \( \Omega \) for the ordinary waves is then the equations for an unmagnetized plasma. The dispersion relation vanishes and the set of equations is reduced to the equivalent are:
\[ \omega^2 = \frac{c^2 k^2}{\omega^2} + \omega_e^2, \]
\[ n_o^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_e^2}{\omega^2}. \]
And these waves propagate as if there were no magnetic field.

2) X-mode, Extraordinary waves:
Now suppose the electric field \( \mathbf{E}_1 \) is in the \( x-y \) plane, perpendicular to \( \mathbf{B}_0 \), taking the form \( \mathbf{E}_1 = E_x \hat{x} + E_y \hat{y} \). This electric field will induce an electron velocity in its direction and the \( \mathbf{v}_e \times \mathbf{B}_0 \) force will produce another velocity component also in the \( x-y \) plane. The dispersion relation for the extraordinary mode is
\[ \left( 1 - \frac{\omega_e^2}{\omega_0^2} \right) \left( 1 + \frac{k^2 c^2}{\omega_0^2} - \frac{\omega_e^2}{\omega_0^2} \right) + \frac{\Omega_e^2 k^2 c^2}{\omega_0^4} - \frac{\omega_0^2 \Omega_e^2}{\omega_e^4} = 0. \]
Where \( \Omega_e \) is the electron gyrofrequency, which is dependent on the magnetic field \( \mathbf{B}_0 \).
\[ \Omega_e = \frac{e B_0}{m_e c}. \]
The dispersion relation for this mode is
\[ n_X^2 = \frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_e^2}{\omega_0^2} - \frac{\omega_0^2 - \omega_e^2}{(\omega_e^2 + \Omega_e^2)^2}. \]
In this mode the wave is partially transverse and partially longitudinal. By solving the set of equations it can be shown that the components \( E_x \) and \( E_y \) are out of phase with each other, resulting in an electric field that performs an elliptical rotation as a function of time at a certain point in space.

The extraordinary mode has two relevant properties, the cutoffs and resonances. A cutoff frequency \( \omega \) occurs when \( k \to 0 \), resulting in the zeros of the index of refraction in (20). The resonance is any frequency where the wavenumber becomes \( k \to \pm \infty \), which is when the denominator of (20) is zero.

The resonances of the X-mode appear for \( \omega = 0 \) and \( \omega = \omega_{1/2} = \omega_{1/2} = \sqrt{\omega_e^2 + \Omega_e^2} \). The cutoffs are obtained by setting (20) equal to zero. Two cutoffs \( \omega_R \), and \( \omega_L \) are obtained from [8]
\[ \omega_{(L,R)} = \pm \frac{\Omega_e}{2} + \sqrt{\omega_e^2 + \Omega_e^2 / 4}. \]

Fig. 2 shows the dispersion diagram for both O and X modes. It is relevant to mention that, considering a real \( \omega \), the regions where \( n < 0 \) correspond to an imaginary wavenumber and to an evanescent wave. These regions are called stop bands as the waves cannot propagate. The other regions are called pass bands and the wave can propagate freely.

### III. Microwave Reflectometry Diagnostics

The principle of operation for reflectometry diagnostics has been widely used as ionosphere plasma diagnostics. Its use to measure electron plasma density in laboratories was first suggested in 1961 [9]. However, only after recent technological advances was reflectometry considered as an important diagnostic for fusion plasmas. Microwave reflectometry is based on the fact that the electromagnetic waves propagating in a plasma are reflected at a critical density layer where the refractive index goes to zero. This happens when the probing wave frequency is equal to the local plasma layer frequency, in O-mode. By having a microwave antenna emitting waves and a receiver it is possible to estimate the plasma density from the frequency of the probing wave. Measuring the time delay between the emission and reception, enables the estimation of the distance between the diagnostics and that plasma layer. By combining both methods and sweeping the frequency of the probing wave, the density profile in the peripheral and scrape-off layer plasma inside a tokamak vessel can be obtained. This sweepable frequency technique uses a frequency-modulated continuous wave with a sawtooth shape to probe the whole frequency range.

Besides swept narrow or broadband techniques, several other have been developed such as fixed frequency, amplitude modulation of a continuous wave, pulse radar, pulse compression, noise correlation. These techniques try to overcome
problems like plasma turbulence or spurious effects due to the transmission lines.

For reflectometry diagnostics, the fusion plasma inside a tokamak can be described by some approximations. In the absence of electromagnetic perturbations, the plasma particles oscillate around their equilibrium positions (cold plasma approximation) and the plasma is homogeneous in all directions except in the direction of the propagation of the wave (slab model). The plasma is considered to be a fluid of ions and electrons coupled to the electromagnetic waves (fluid approximation). The frequency of the electromagnetic wave is much higher than all the plasma ion cyclotron and ion plasma frequencies (high frequency approximation).

As seen in Fig. 1 the sheared magnetic field inside a tokamak is composed of toroidal $B_\phi$ and poloidal $B_\theta$ components. It is interesting to notice that the poloidal component is negligible compared to the toroidal component ($B_\phi \gg B_\theta$) and so the magnetic field is mainly toroidal $B_\phi \approx B_\phi$. For this reason, emitting waves that propagate in the O-mode have their electric field aligned with the toroidal direction of the tokamak and the magnetic field, and the refractive index is independent of the applied magnetic field, simplifying plasma density profile inversion from reflectometry data.

The basic configuration of a reflectometer consists of an oscillator, a detector and the transmission lines to guide the signal from the oscillator to the emitting antenna and from the receiving antenna to the detector, as presented in Fig. 3 [10].

In one configuration, the transmitting signal is sampled by a reference pin, which is used as a reference signal that is mixed with the plasma reflected wave. The measured signal at the output of the detector depends on both phase and amplitude of the reflected signal, leading to ambiguity. Another topology combines the plasma reflected wave signal and the reference signal in a mixer, resulting in a beat frequency signal proportional to the phase shift in the reflected signal. Both of these detection methods are presented in Fig. 3. A more advanced technique is the quadrature-phase detection. Mixers are used and the reference signal is split into two parts, one with an extra $\pi/2$ phase shift. These quadrature reference signals are combined into two mixers which are also fed the split plasma reflected signal. The mixer output allows obtaining the absolute power and absolute phase of the plasma reflected wave. However, this technique is limited to the introduced phase shift being valid only for one frequency. A heterodyne configuration allows increasing the dynamic range of the detection but requires a considerable time to recover from large phase jumps. To probe the complete density profile, a broadband sweepable source or several fixed frequency sources are used.

In active probing, the diagnostics probing waves have a low amplitude and their perturbations introduced in the plasma are negligible. Reflectometers for density profile measurement have been designed with frequencies up to 110 GHz [11], measuring densities up to $1.5 \times 10^{20} m^{-3}$, in both O [12] and X [13] propagation modes.

Due to the complexity of the refractive index of the X-mode propagation, it is not possible to analytically invert the propagation path of the wave. Numerical algorithms must be used to recover the position of the reflecting layer. This mode of operation is limited as the process depends on the correct knowledge of the magnetic field. However, if the density profile is known, the X-mode can be used to derive the magnetic field profile. In this section, only the O-mode propagation is considered.

A. Measurement principle

The microwave reflectometer works by launching a wave with a frequency $f$ to the plasma, propagating through it as long as the plasma frequency, or cutoff frequency, is lower than the wave frequency. This means that the wave propagates through the increasing density regions of the plasma up to the critical density plasma layer where the wave frequency matches that of the plasma frequency. Here, the refractive index becomes purely imaginary and the wave is reflected. The wave suffers a total phase shift on the round-trip that is equivalent to the time delay of a returning radar echo. The temporal variation of the phase $\partial \phi / \partial t$ can result either from the variation of the probing wave frequency or of the optical path length between the antenna position $x = x_0$ and the cutoff reflecting layer $x = x_{co}$. Consider the antenna aligned with the tokamak so that the line of sight is into the plasma towards the centre of the plasma column and the position $x$ is aligned radially. Other reflectometer antenna configurations may exist.

The local refractive index of the plasma for the ordinary mode varies depending on the probing wave and on the position of the wave in the plasma $n_0(f,x)$. The optical path length $L_{op}$ up to a cutoff layer position is given by integrating the ordinary mode refractive index $(n_0)$ along the propagation path.

$$L_{op} = \int_{x=0}^{x_{co}} n_0(f,x) dx \quad (22)$$

Maxwell equations do not lead to an exact solution of electromagnetic wave propagation in inhomogeneous plasmas. Approximate solutions of the wave equation are usually obtained to extract the incident and reflected wave contributions. One of these solutions is the Wentzel-Kramers-Brillouin (WKB) approximation [14] that corresponds to neglecting the secondary multiple reflections occurring at each infinitesimal small width plasma layer. This approximation is valid if the probing wavelength is smaller than the density gradient’s characteristic length $L = n/|dn/dr|$ [15]. As insight, consider a characteristic length of $L \approx 3 cm$, near the plasma edge; the WKB solution is valid for probing waves with $\lambda \leq 3 cm$, corresponding to a probing frequency of $f \geq 10 GHz$ and a
detectable plasma density above \( n_e \approx 0.12 \times 10^{19} \text{ m}^{-3} \) [10]. The temporal variation of the phase can result either from the variation of the probing frequency or of the optical path length between the antenna and the reflecting layer, and is given by

\[
\frac{\partial \phi}{\partial t} = \frac{4\pi}{c} \left( \frac{\partial f}{\partial t} \right) \int_{x=0}^{x=0} n_0(f, x) dx + \frac{4\pi}{c} f \frac{\partial}{\partial t} \left( \int_{x=0}^{x=0} n_0(f, x) dx \right).
\]

(23)

The position of the reflecting layer can be determined using the first term on the right hand side of (23) by sweeping the probe frequency, measuring \( \partial \phi / \partial f \) and the corresponding wave group delay

\[
\tau_g(f) = \frac{1}{2\pi} \frac{\partial \phi}{\partial f} = \frac{1}{2\pi} \frac{\partial}{\partial t} \left( \frac{\partial f}{\partial t} \right)^{-1}.
\]

(24)

Where \( \partial \phi / \partial f \) is the interference signal phase derivative and \( \partial f / \partial t \) is the probing frequency rate. The second term on the right hand side of equation (23) describes the phase variations due to the changes of the optical path length that arise from temporal and spatial variations of the plasma electron density. By fixing the probing frequency, the measurement of the phase variations of the reflected signal can be used to monitor the density changes and the turbulent fluctuations of the plasma. The wave phase deviations that result from the plasma turbulence may be avoided by sweeping the probe frequency on a time scale expected to be shorter than the time scale of the most significant motions of the cutoff layers \((\leq 100\mu s)\). The phase shifts are attenuated because the phase measurements are made on a quasi frozen plasma.

Considering only the first term of (23), the position of the cutoff layer \( x_{co}(f_{co}) \) for a probing frequency \( f_{co} \) can be recovered analytically by performing an Abel inversion

\[
x_{co}(f_{co}) = X_0 \pm \frac{c}{\pi} \int_{f_0}^{f_{co}} \frac{\tau_g(f)}{\sqrt{f_{co}^2 - f^2}} df.
\]

(25)

Provided that \( \tau_g(f) \) is known with sufficient accuracy to perform the integration. \( X_0 \) is the reference position, between the antennas and the first plasma density, that corresponds to \( \tau_g(f) = 0 \), so that every measurement has a common reference (+ for the high field size; - for the low field size of the vessel).

### B. Density profile reconstruction

The reconstruction of the plasma electron density profile requires the correct determination of the group delay. To probe the full density gradient region, a wide frequency range must be scanned. However, this means that the frequency range must be divided into several waveguide bands, requiring that several reflectometers, each connected to its own waveguide band, have to be used simultaneously so that the full frequency range is covered. Also, the lowest frequency range cannot be probed as the probing wavelength becomes too large for the local density gradient scale length and the WKB approximation condition is not valid. The group delay evolution in this low frequency range must be estimated, which may lead to errors in profile recovery. Physically, this low frequency region corresponds to the low plasma electron density region from vacuum at the vessel walls, to the first plasma layer with a measurable density. The microwave reflectometry O-mode group delay can be initialised with data from other density diagnostics with relevant data on the profile edge, with X-mode data [10], or using a model for the initial part of the plasma edge. Such a model can use as input the measured group delay and extrapolate the curve down to zero exponentially [16]. Other group delay initialisations include real data, linear extrapolation or using the vessel limiters as the \( X_0 \) position.

After knowing the main aspects required for the reconstruction of the density profile, one can then consider the necessary signal processing. The signal at the output of the detector has a DC term dependent on the amplitudes of the probing and reflected and a term with a beat frequency, which is dependent on the amplitudes and also on the cosine of the phase shift of the reflected wave, as

\[
s_d(t) = \frac{a_p(t)^2 + a_r(t)^2}{2} + a_p(t)a_r(t)\cos(\phi(t)) + n_d(t)
\]

(26)

The noise contribution term, resulting from all noise sources, such plasma emission or plasma heating, is also included. The first step to extract the phase shift \( \phi(t) \) from \( s_d(t) \) is to eliminate the low frequency and the noise components. This is achieved by filtering either in hardware or in the digital domain. The latter has the advantage of being dynamic, having a higher performance and steeper filter boundary curves without adding more noise with each stage, nor distorting the phase of the signal, at the cost of increased CPU time.

The beat frequency obtained at the detector \( f_0(t) \) is related to the interference signal phase derivative [5], [17] and the group delay as

\[
f_0(t) = \frac{1}{2\pi} \frac{\partial \phi}{\partial t} = \frac{1}{2\pi} \frac{\partial}{\partial f} \frac{\partial f}{\partial \tau} = \frac{\partial f}{\partial \tau} \tau_g(f)
\]

(27)

Where the probing frequency rate is a constant after linearisation of the sweep generation. The beat frequency is the instantaneous frequency [18] and evaluating \( \tau_g(f) \) now becomes a frequency estimation problem. Different techniques are available for frequency estimation: fringe counting [19], zero-crossing, minima/maxima [20], time-based frequency discriminator [21] and complex demodulation [22], and time-frequency spectral analysis.

The calculation of the time-frequency distribution (TFD) of the energy density of the interference signal allows both the inspection of the time evolution of the signal’s spectral content and the computation of its instantaneous frequency. The performance of this analysis depends on the trade-off between time and frequency resolutions required to build the TFD. One of the main advantages of using spectrograms is that they allow the use of the frequency domain information in adjacent swept bands in order to estimate the beat frequency. By building a combined distribution of these adjacent bands, it may be possible to find a beat frequency evolution that better matches the mean density profile as the locally perturbed beat frequency signals are attenuated. A complete TFD of all the measured bands requires the calculation of the individual band spectrogram and merging all the TFDs together, as is presented in Fig. 4.
The extraction of the beat frequency from the spectrogram is one of the most critical steps in the computation of the density profile. For a low turbulence plasma, the interference signal is concentrated around the beat frequency. However, as plasma turbulence increases, the interference signal energy is scattered over a wider frequency range, not necessarily over the central beat frequency which corresponds to the underlying unperturbed density profile. The beat frequency estimation can be obtained from the spectrogram by the detection of the maximum peak, the calculation of the first moment or the computation of the best path.

The density profile along the radius of the vessel is calculated by performing the Abel inversion of (25). This inversion integral can be decomposed into two sub-integrals $A_1$ and $A_2$

$$R_{co}(f_{co}) = \begin{cases} \frac{\pi f_{co}}{2} \int_{f_{co}}^{f_{co}} \frac{\tau_{g init}(f)}{\sqrt{f_{co}-f^2}} df = A_1 \\ \frac{\pi f_{co}}{2} \int_{f_{co}}^{f_{co}} \frac{\tau_{g init}(f)}{\sqrt{f_{co}-f^2}} df + \frac{\pi f_{co}}{2} \int_{f_{co}}^{f_{co}} \frac{\tau(f)}{\sqrt{f_{co}-f^2}} df = A_2 \end{cases} \quad (28)$$

The term $A_1$ accounts for the contribution of the group delay. $\tau_{g init}(f)$ corresponds to an estimation of the initial part of the profile that is not probed. The term $A_2$ accounts for the contribution of the measured group delay obtained by probing frequencies above $f_{co0}$, which is the first probing frequency of the O-mode.

The standard reflectometry O-mode automatic density profile measurement procedure in ASDEX Upgrade [5] consists of: Acquiring reflectometry interference data sweeps per channel; Linearising and pre-filtering the interference signals; Producing a combined spectrogram with the burst measurements; Extract the group delay using the best path algorithm; Initialising the group delay data to a fixed radial position; Inverting the profile using the discrete Abel integration. Even though the procedure is automatic and deterministic, human supervision is needed to validate or discard the final profile data.

C. Typical reflectometry density profile results

Presently, there are no diagnostics capable of producing absolute electron density profile data with time and space resolutions that allow it to be used as reference or basis of comparison with other diagnostics. In Fig. 5 the effects that a type-I ELM has on the density profile obtained with reflectometry measurements in the high (HFS) and low (LFS) field sides is presented. It is observed that the impact of the ELM on the density profile on the periphery of the HFS is much more dramatic than on the LFS and the recovery time is much shorter in the LFS. This is consistent with the measured $D_\alpha$ radiation in the divertor.

IV. Conclusion

In this paper the theory of electromagnetic wave propagation in unmagnetised and magnetised plasmas was presented. Depending on the angle that the electric field of the wave has with the magnetic field, the waves can be propagating in ordinary and extraordinary modes. The wave O-mode propagation was shown to be independent of the magnetic field.

The typical microwave reflectometry topology and the principle of measurement of plasma density profiles and plasma turbulence were explained. The emitted waves propagate in the plasma and are reflected by the critical density plasma layers where the plasma frequency equals that of the wave. It was seen that aligning the emitting antennas so that the waves propagate in O-mode, allows density measurement without local magnetic field information.

The density profile can be reconstructed from the reflectometry interference data. Plasma turbulence near the critical layers affect the extraction of the beat frequency from the measurements and data processing algorithms must be implemented, depending on the physics expected. Experimental results obtained with a microwave reflectometry system at ASDEX Upgrade were presented.

More advanced techniques exist both in the reflectometry system design, considering other reflectometry topologies, and in the data processing. Reflectometry systems can be used for offline density profile and plasma turbulence measurements but have also been used for real-time measurements and also in the feedback loop of the plasma position control. Future work in reflectometry diagnostics include improving these systems to be used in real-time plasma position control, as has been shown to be possible.
REFERENCES


